(Pages : 3) **3043**

Reg. No. : ....

Name : ....

Third Semester B.Tech. Degree Examination, April 2015
(2013 Scheme)
13.303 : DISCRETE STRUCTURES (FR)

Time: 3 Hours Max. Marks: 100

PART – A

Answer **all** questions. **Each** question carries 2 marks.

1. Write an equivalent formula for $\urcorner(P \leftrightarrows (Q \to (R \vee P)))$ and which contains the connectives $\wedge$ and $\urcorner$ only.

2. Negate the following statement and represent in symbolic form :
"Some apples are red".

3. Show the validity of the following :
$(\urcorner(P \wedge \urcorner Q)) \wedge (\urcorner Q \vee R) \wedge \urcorner R \Rightarrow \urcorner P$.

4. Show that if any 5 integers from 1 to 8 are chosen, atleast two of them will have a sum 9.

5. Using mathematical induction. Prove that $n < 2^n$.

6. Give the conditions that a relation must satisfy in order to qualify as a function.

7. Let $(A, \cdot)$ be a group. Show that $(ab)^{-1} = b^{-1}a^{-1}$.

8. Prove that every cyclic group is Abelian.

9. What is meant by connected components ? Explain.

10. Prove that the zero element and unit-element of a Boolean algebra 'B' are unique.

(10×2=20 Marks)

P.T.O.

PART – B

Answer **any one** question from **each** Module.

Module – I

11. a) Show that $(x)\,(P(x) \vee Q(x)) \Rightarrow (x)\,P(x) \vee (\exists x)\,Q(x)$ using indirect method of Proof. **10**

b) "If there was a meeting, then catching the bus was difficult. If they arrived on time then catching the bus was not difficult. They arrived on time. Therefore, there was no meeting". Check the validity of the argument. **10**

12. a) Show the following without constructing the truth tables :
$\neg(P \wedge Q) \to (\neg P \vee (\neg P \vee Q)) \Leftrightarrow (\neg P \vee Q)$. **5**

b) Derive the following, Using CP if necessary :

$P \to (Q \to R), Q \to (R \to S) \Rightarrow P \to (Q \to S)$. **10**

c) Discuss the different types of quantifiers used in predicate logic. **5**

Module – II

13. a) For any two sets A and B, show that $A - (A \cap B) = A - B$. **5**

b) Using Mathematical induction, prove that $6^{n+2} + 7^{2n+1}$ is divisible by 43. **10**

c) Differentiate between a partition and covering of a set with examples. **5**

14. a) Given the relation matrices M_R and M_S. Find $M_{R \circ S}$, $M_{\tilde{R}}$, $M_{R \tilde{\circ} S}$ and show that $M_{R \tilde{\circ} S} = M_{\tilde{S} \circ \tilde{R}}$

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad M_S = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}.$$ **10**

b) Draw the Hasse diagram of $(P(A), \leq)$ where $\leq$ represents $A \subseteq B$ and $A = \{a, b, c\}$. **5**

c) What are Peano Axioms ? Explain. **5**

Module – III

15. a) Show that $(I, \oplus, \odot)$ is a commutative ring with identity, where the operations $\oplus$ and $\odot$ are defined for any $a, b \in I$ as

$a \oplus b = a + b - 1$ and

$a \odot b = a + b - ab$. **10**

b) Let $(H, \cdot)$ be a subgroup of a group $(G, \cdot)$. Let $N = \{x \mid x \in G, xHx^{-1} = H\}$. Show that $(N, \cdot)$ is a subgroup of $(G, \cdot)$. **10**

16. a) Show that any subgroup of a cyclic group is cyclic. **10**

b) Prove that every field is an integral domain. **10**

Module – IV

17. a) In a distributive lattice $a \wedge b = a \wedge c$ and $a \vee b = a \vee c$ imply that $b = c$. **10**

b) Let $(L, \leq)$ be a lattice and a, b, c, d, $\in$ L. Prove that if $a \leq c$ and $b \leq d$ then

i) $a \vee b \leq c \vee d$

ii) $a \wedge b \leq c \wedge d$. **10**

18. a) Differentiate between reachability and connectedness with examples. **7**

b) Prove that the complement of every element in a Boolean Algebra is unique. **3**

c) Prove that in a distributive lattices if $b \wedge c = 0$; then $b \leq c$. **10**
